KIT Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
High Performance Humanoid Technologies (H^2T)

Robotics I, WS 2024/2025

Exercise Sheet 4

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Date: January 8, 2025

Exercise 1

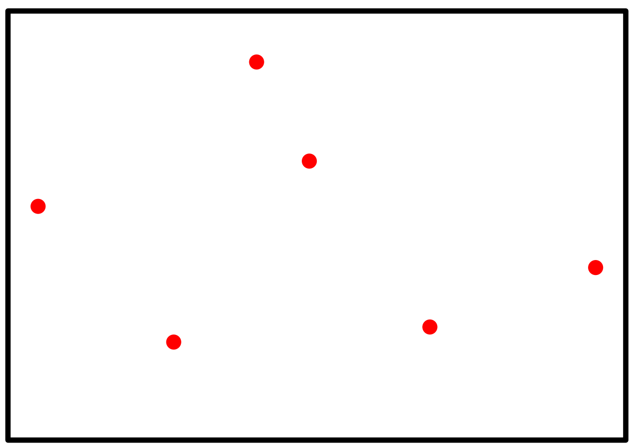


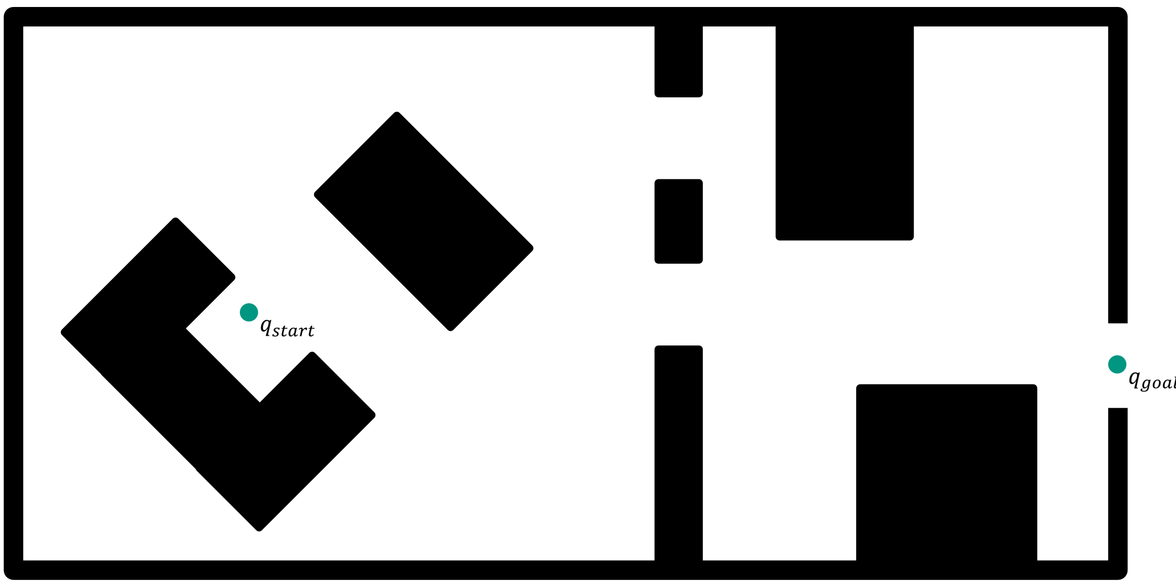
Figure 1: The point set P.

Let P be a set of points shown in Figure 1.

1. Explain the terms Voronoi region, Voronoi edge and Voronoi vertex.
2. Determine the Voronoi diagram for P.

Exercise 2

The following floor plan of an apartment is given:



1. Determine the cell decomposition of the given floor plan using the Line-Sweep technique and number the cells.
2. Draw the adjacency graph of the resulting cells.
3. Determine a path from q_{start} to q_{goal} and determine the sequence of the cells passed through.

Exercise 3

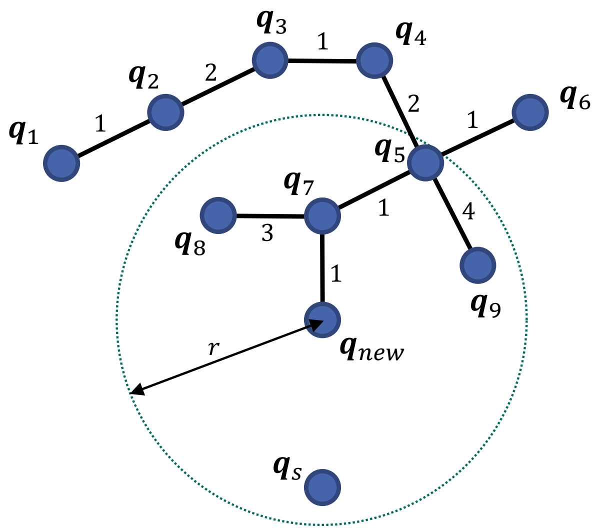


Figure 2: The RRT* tree T.

The tree T shown in Figure 2 shows an intermediate step of the RRT* algorithm. The nodes are labeled q_1 to q_9 and the costs of the connections are indicated on the edges. In the current iteration step, the node q_{new} was added to the tree.

1. Explain how the node q_{new} was determined.
2. Calculate the path costs for the nodes $q_1, \ldots, q_9, q_{new}$.
3. Describe the RRT* function $\text{Near}(T, q_{new}, r)$.
4. Which nodes are taken into account for the *Rewire* step of the RRT* algorithm? Justify your answer.
5. Draw the connections after the *Rewire* step in Figure 3. Consider the following costs:

   $\text{Cost}(q_{new}, q_5) = 5$, $\text{Cost}(q_{new}, q_8) = 1$, $\text{Cost}(q_{new}, q_9) = 1$.

   Please justify your result.

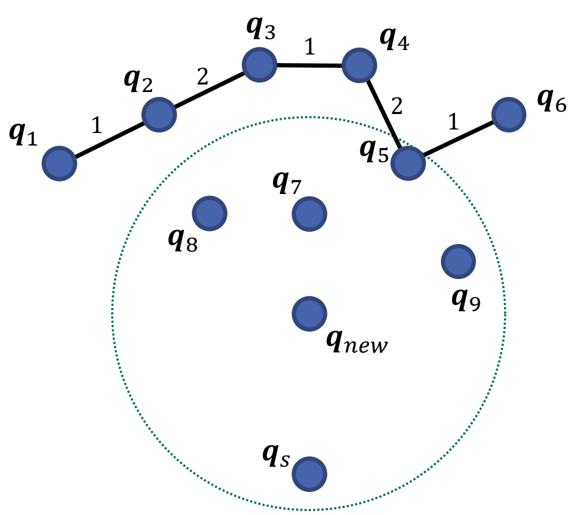
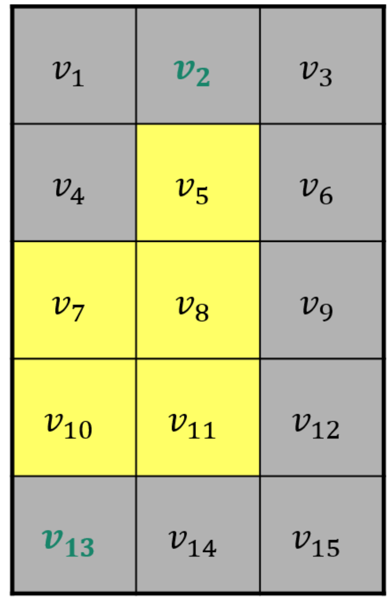


Figure 3: Draw the connections after the RRT*-*Rewire* step.

Exercise 4

In the grid shown on the right, the shortest path between v_2 and v_{13} should be determined using the A* algorithm. The following conditions apply:

1. Only horizontal and vertical movements to neighboring nodes are permitted. The distance between the nodes is assumed to be 1.
2. The cost to move to a node depends on the color of the node (gray nodes: 1, yellow nodes: 4).
3. The Euclidean distance to the target node v_{13} is used as the heuristic h (e.g., $h(v_{11}) = \sqrt{2}$).



Please complete the following subtasks:

1. Specify the first three steps of the A* algorithm. Please indicate the node to be expanded and the changes to the node sets (Open Set, Closed Set).
2. Why is the Euclidean distance a suitable heuristic in this task?
3. When does the A* algorithm find a valid solution?
   (a) When the next node to be expanded is the target node.
   (b) When the target node is added to the Open Set.

   Please justify your answer.

Exercise 5

Let R be a point-shaped moving robot at the position $\mathbf{q}_R \in \mathbb{R}^2$ and let there be three repulsive potentials $U_{\text{rep},1}$, $U_{\text{rep},2}$ and $U_{\text{rep},3}$ at the positions $\mathbf{q}_{\text{rep},1}$, $\mathbf{q}_{\text{rep},2}$ and $\mathbf{q}_{\text{rep},3}$. Each potential represents a point-shaped obstacle. In addition, there is an attracting target potential U_{attr} at the position $\mathbf{q}_{\text{goal}}$. The positions of the potentials are

$$\mathbf{q}_R = \begin{pmatrix} 5 \\ 5 \end{pmatrix}, \quad \mathbf{q}_{\text{rep},1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad \mathbf{q}_{\text{rep},2} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \quad \mathbf{q}_{\text{rep},3} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \quad \mathbf{q}_{\text{goal}} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}.$$

1. Which of the repulsive potentials $U_{\text{rep},1}$, $U_{\text{rep},2}$ and $U_{\text{rep},3}$ act on the robot, assuming a radius of influence of $\rho_0 = 5$?
2. Determine $U(\mathbf{q}_R)$ as the sum of the acting potential fields. Assume that the repulsive potentials are described by a FIRAS function with $\nu = 1$ and that the target potential is described by a linear function with $k = 1$.
3. Determine the direction in which the robot would move due to the acting potentials.